International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN (P): 2319-3972; ISSN (E): 2319-3980
Vol. 9, Issue 2, Feb–Mar 2020; 9-20

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

MORTALITY ANALYSIS IN HETEROGENEOUS POPULATIONS

Talawar A. S[1] & Rajani P. A[2]

[1]Professor, Department of Statistics, Karnatak University, Dharwad, India

[2]Research Scholar, Department of Statistics, Karnatak University, Dharwad, India

ABSTRACT

In the mortality data, a lower order polynomial does not provide a good fit especially in case of age interval. A possible approach to get a good fit is to increase the order of the polynomial. The higher order polynomial works well for mortality data with age interval of five years and suitable when mortality data for single year is not available. We use the polynomial regression model in one explanatory variable to fit mortality data where mortalities are available in age interval. In case of the higher order polynomials, the problem of multicollinearity is resolved by centering explanatory variable. We observe from the fitting that the polynomial regression model is very good approximation for all the three heterogeneous subpopulations. For all the subpopulations (male & female, rural male & female and urban male & female) polynomial approximation is the simplest suitable choice of fitting model to mortality data. Using the estimated mortality values by age interval, other columns of life tables are constructed. Life expectancies for these subpopulations are presented in the tables.

KEYWORDS: *Mortality, Polynomial Regression, Heterogeneity, Co-Efficient of Determination, Life Expectancy*

Article History

Received: ***30 Jan 2020*** **|** ***Revised:*** ***07 Feb 2020*** **|** ***Accepted:*** ***18 Feb 2020***

INTRODUCTION

There is a vast literature on stochastic modelling of mortality rates. The term stochastic model refers to the model which expresses mortality-related quantities such as the mortality rate, the force of mortality or the number of survivors as functions of age. Consequently, these models describe the age-specific mortality rates of certain cohorts or certain periods. The age and time dependent models of mortality are described by Lee and Carter (1992). Many authors have contributed to modelling age-specific mortality rates (Lee and Miller, 2001; Booth et al., 2002; Brouhns et al., 2002; Renshaw and Haberman, 2003; Currie et al., 2004; Currie, 2006; Renshaw and Haberman, 2006 and Cairns et al., 2006). In these papers a mortality model was proposed aiming at combining the characteristics from or eliminating the disadvantages of the existing models. Stochastic mortality models either model the central mortality rate or the initial mortality rate (Coughlan et al., 2007). Due to the increasing focus on risk management and measurement for insurers and pension funds, the literature on stochastic mortality models has developed rapidly during the last decennium (Booth and tickle, 2008).

Stochastic models that express mortality as a function of age, or those that take into account both the age and time dependency of mortality, can use various assumptions to model out all possible observed characteristics of mortality patterns. One of the most commonly used considerations is that the population is heterogeneous and composed of several subpopulations having different mortality dynamics (Rossolini and Piantanelli, 2001). Tabeau et al. (2001) and Vaupel

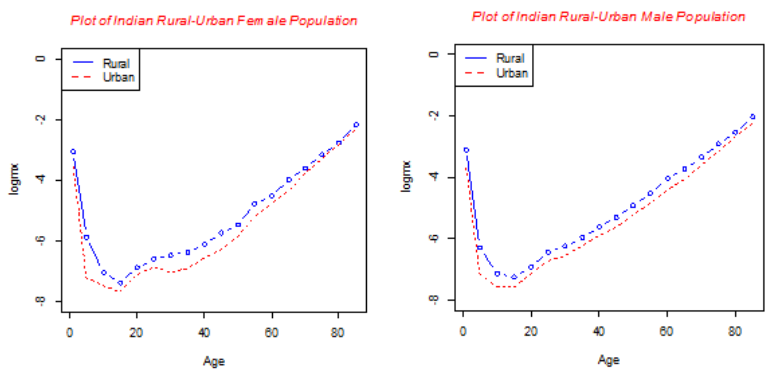
(2005) presented the age-dependent mortality models which are separated into groups and expressed by polynomial and non-polynomial functions (see also Keyfitz and Littman, 1979 and Keyfitz, 1984). Brouhns et al., (2002) described a fitting methodology for the Lee-Carter model based on a Poisson model. The main advantage of this is that it accounts for heteroskedasticity of the mortality data for different ages. The authors contributed towards this are Renshaw and Haberman, (2003, 2006); Cairns et al. (2007) and Cairns et al. (2011). Many functions can be approximated by polynomials using Taylor expansions (Vaupel, 2010; Tomas, 2012) and this gives an advantage in modelling mortality-related data.

The mortality patterns in human populations reflect biological, social and medical factors affecting our lives, and stochastic modelling is an important tool for the analysis of these patterns. It is known that the mortality rate in all human populations increases with age after sexual maturity. This increase is predominantly exponential and satisfies the Gompertz pattern. Although the exponential growth of mortality rates is observed over a wide range of ages, it excludes early and late-life intervals. In a model of heterogeneous populations Avraam, et al. (2015) studied how differences in parameters of the Gompertz equation and described different subpopulations accounting for mortality dynamics at different ages.

MORTALITY IN HETEROGENEOUS POPULATIONS

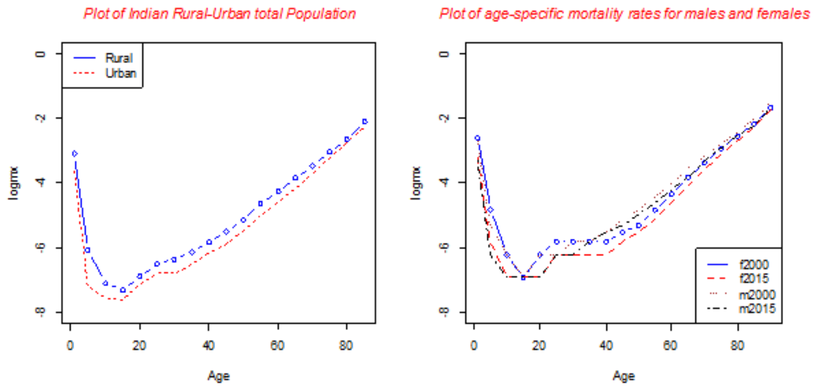
In the present paper, we apply polynomial regression model to fit the mortality data of various subpopulations. The Least Squares (LS) method is used to estimate the free (unknown) parameters that minimize the sum of squared residuals. The LS method overestimates mortality at early life and underestimates it at old ages when simple functions such as the Gompertz, Makeham or Weibull are used to fit the mortality rates. The LS method is favoured over many other methods because of its simplicity (provided the assumptions of regression model are satisfied).

Each human population can be observed as number of subpopulations which differ genetically (sex-male and female) and/or by place of residence (rural and urban) and/or ethnicity (black, white, etc). Therefore, we can model the whole heterogeneous population in the following way. We consider a population consisting of two heterogeneous subpopulations (males & females, rural males & females and urban males & females). The mortality of each subpopulation is described by polynomial regression with parameters specific to the subpopulation. Figures (a-j) show the differences in mortality rate among the subpopulations. The mortality for all the subpopulations increases at young ages, decreases for a short age interval and then increases again for old ages.



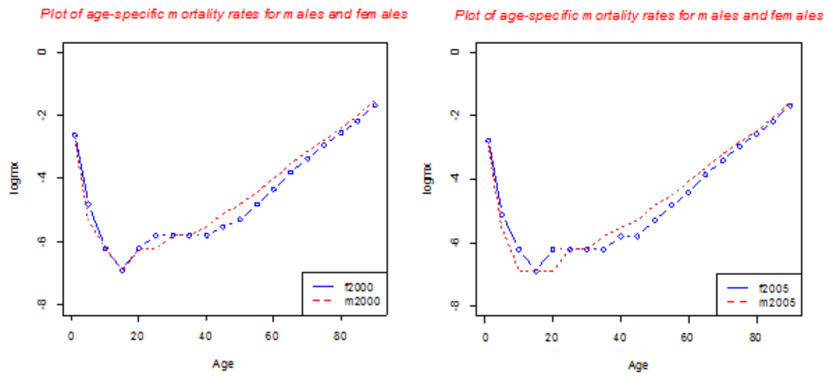
(a) (b)

Figures (a-b): Differences in the Observed Age-Specific Mortality Rates (in log scales) for (a) India Rural and Urban Female Population between the Years 2012-16 and (b) India Rural and Urban Male Population between the Years 2012-16.



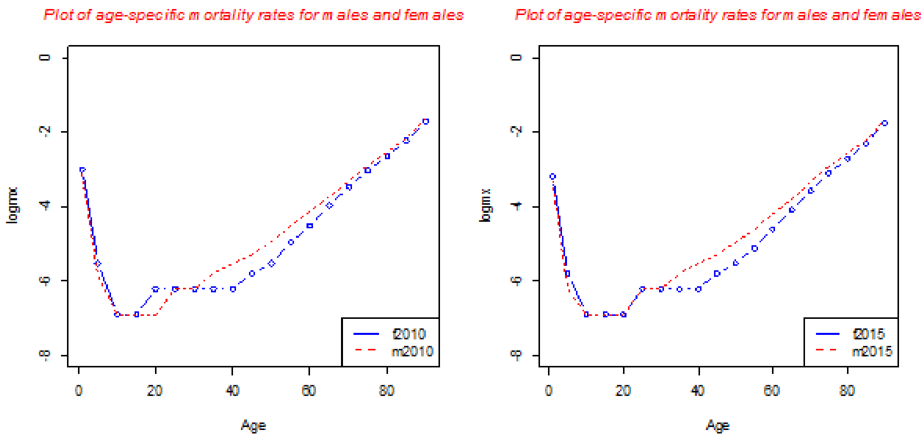
(c) **(d)**

Figures (c-d): Differences in the Observed Age-Specific Mortality Rates (in log scales) for (c) India Rural and Urban Total Population between the Years 2012-16 and (d) India Males and Females between the Years 2000 and 2015.



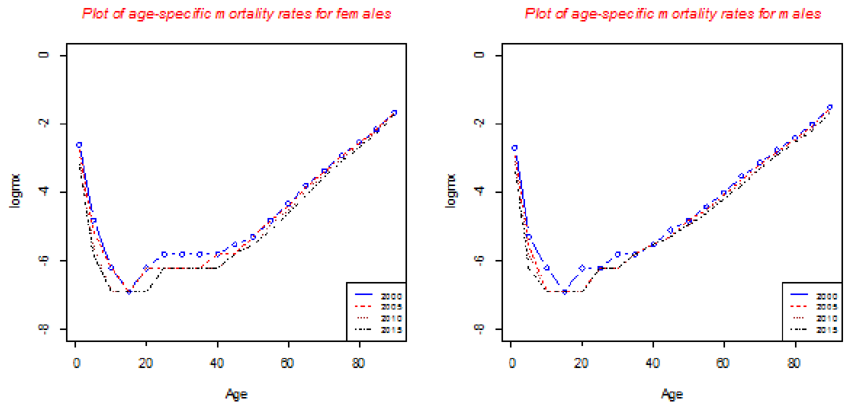
(e) **(f)**

Figures (e-f): Comparison of Observed Age-Specific Mortality Rates (in log scales) for (e) India Males and Females of the Year 2000 and (f) India Males and Females of the Year 2005.



(g) **(h)**

Figures (g-h): Comparison of Observed Age-Specific Mortality Rates (in log scales) for (g) India Males and Females of the Year 2010 and (h) India Males and Females of the Year 2015.



(i) **(j)**

Figures (i-j): Differences in the Observed Age-Specific Mortality Rates (in log scales) for (i) India Females and (j) India Males during the Years 2000, 2005, 2010 and 2015.

(Source:http://www.censusindia.gov.in/VitalStatistics/SRSLifeTable/SrslifeTable2012-16.html. https://www.who.int/healthinfo/mortality_data/en/)

Logarithms of the mortality rates for India males, females, rural and urban are used for the analysis. Figures (i-j) show the mortality pattern of females and males for four years (2000, 2005, 2010 and 2015). The similarity in shape over the years of both the male and female mortality curves is the general downward trend of mortality. Also some changes over time in the shape of the curves can be observed particularly the increasing importance of the accident hump, the sharp increase in mortality at ages 15-19, in both the male and female mortality rates figures (a-j).

POLYNOMIAL APPROXIMATION TO MORTALITY DATA

For the mortality analysis, we consider mortality of different subpopulations such as male, female, rural male, rural female, urban male and urban female. We fit mortality values using both lower age limit and upper age limit separately as explanatory variable. The fitting for both the cases and for all subpopulations gives good fit. We consider a unique polynomial approximation using these two best fit polynomials. This polynomial approximation is good for all subpopulations except rural male, urban male and female subpopulations.

The polynomial regression model in one variable is given by

$$m_x = e^{\beta_0+\beta_1 x+\beta_2 x^2+\beta_3 x^3+\cdots+\beta_k x^p+e} \tag{1}$$

$$log(m_x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \cdots + \beta_k x^p + \epsilon \tag{2}$$

Thus the techniques for fitting linear regression model can be used for fitting the polynomial regression model. The equation can be expressed in matrix notation as in the case multiple linear regression model

$$.f = X\beta + \epsilon \tag{3}$$

The order of the polynomial model is kept as low as possible (order should be less than the number of observations). A good strategy should be used to choose the order of an approximate polynomial. One possible approach is to successively fit the models in increasing order and test the significance of regression coefficients at each step of model fitting. Keeping the order increasing until -test for the highest order term is non-significant. This is called as forward selection procedure. Another approach is to fit the appropriate highest order model and then delete terms one at a time starting with highest order. This is continued until the highest order remaining term has a significant -statistic. This is called as backward elimination procedure. The forward selection and backward elimination procedures do not necessarily lead to same model (Shalabh, 2018). In the present paper, we use the forward selection procedure.

In the mortality data, a lower order polynomial does not provide a good fit. A possible approach to get a good fit is to increase the order of the polynomial. The higher order polynomial works well for mortality data with age interval of five years and suitable when mortality data for single year is not available. But it will not work especially for data with single year age and will not improve the fit significantly. This type of problems can be addressed by fitting an appropriate polynomial function in different ranges of explanatory variable. So polynomial regression will be fitted into pieces. The spline function can be used for such fitting of regression polynomial in pieces. The piecewise polynomials are called splines. We use the polynomial regression of higher order for the analysis mortality data with age intervals.

In order to have a satisfactory modelling bias, the degree p of the polynomial often has to be large. In the present paper, the highest order of polynomial regression model used is six. The problem of multicollinearity in higher order polynomials can be reduced by centering the explanatory variable.

Model (1) can be written as

$$log(m_x) = \beta_0 + \beta_1(x-\bar{x})^{+} + \beta_2(x-\bar{x})^2 + \beta_3(x-\bar{x})^{3+} + \beta_4(x-\bar{x})^4 + \beta_5(x-\bar{x})^{5+} + \beta_6(x-\bar{x})^6 + \epsilon$$

Or

$$f = \beta_0 + \beta_1 k_1^{+} + \beta_2 k_2 + \beta_3 k_3^{+} + \beta_4 k_4 + \beta_5 k_5^{+} + \beta_6 k_6 + \epsilon \qquad (4)$$

Where $f = logm_x$, $k_i^{+} = \max\{(x-\bar{x})^2, 0\}, i = 1, 3, 5$ and $k_j = (x-\bar{x})^j, j = 2, 4, 6$

Therefore, equation (3) becomes

$$f = A\beta + \epsilon \qquad (5)$$

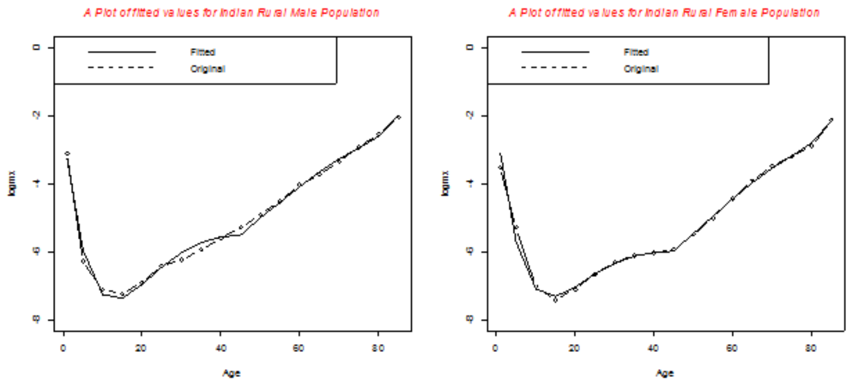
A basic assumption in linear regression analysis is that the matrix A is of full rank matrix. In polynomial regression models, as the order increases, the matrix $A'A$becomes ill-conditioned and then the matrix $(A'A)^{-1}$ may not be accurate and parameters will be estimated with considerable error. The problem of this ill-conditioning is reduced by centering the explanatory variable. For fitting mortality data, we consider a best uniform approximating polynomial which is unique. The following theorem is taken from Celant and Broniatowski (2016, page 6).

Theorem: Let f be a continuous function defined on [a, b]. The best uniform approximating polynomial is unique.

Here, we assume that two such polynomials of best approximation, say P(x) and Q(x) exist. Then

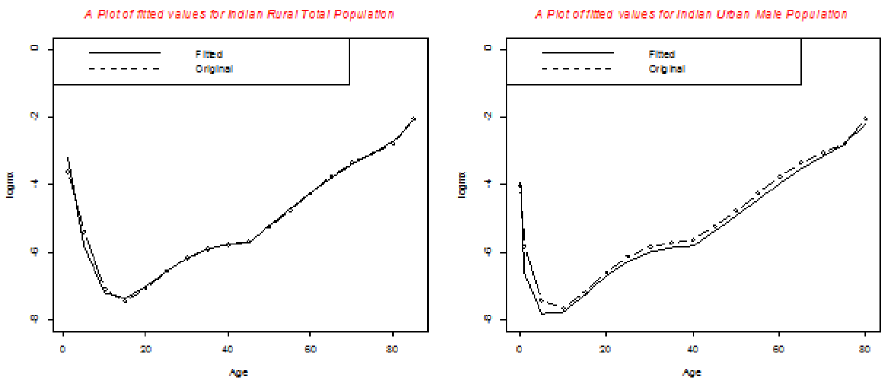
$$S(x) = \frac{P(x)+Q(x)}{2} \qquad (6)$$

By the theorem, it is proved that S(x) is best and unique uniform approximating polynomial. Therefore, we consider the polynomial S(x)is the best uniform approximation of f.



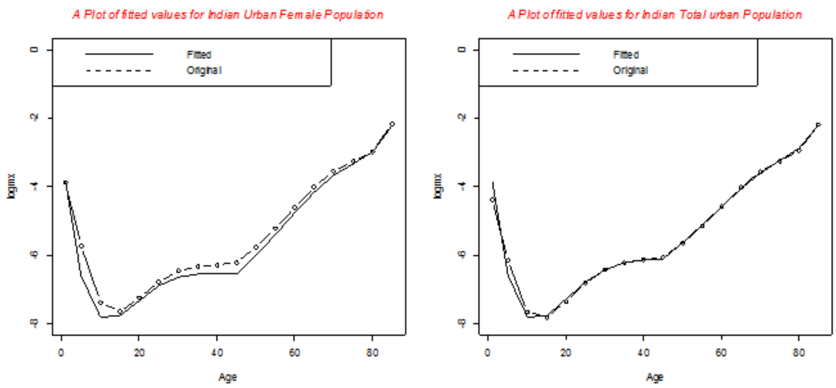
(k) (l)

Figures (k-l): Comparison of Observed and Fitted Values of Age-Specific Mortality (in log scales) for (k) Rural Male (Adjusted R^2=0.9884) and (l) Rural Female (Adjusted R^2=0.992) Population During the Year 2012-16.



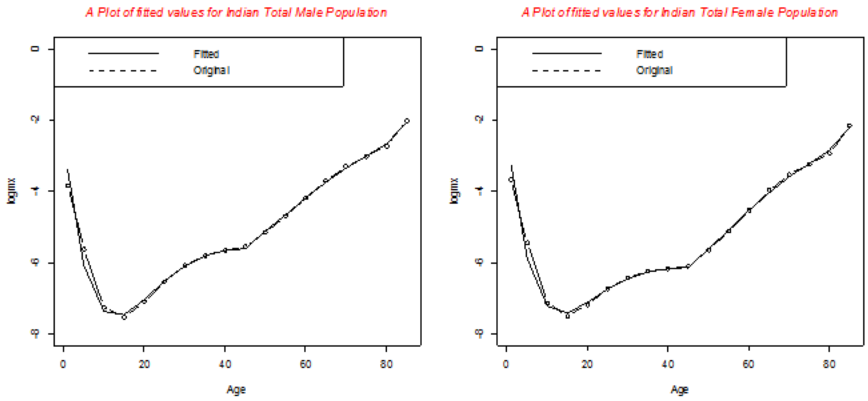
(m) (n)

Figures (m-n): Comparison of Observed and Fitted Values of Age-Specific Mortality (in log scales) for (m) Indian Rural Total (Adjusted R^2=0.9916) and (n) Urban Male (Adjusted R^2=0.9784) Population during the Year 2012-16.



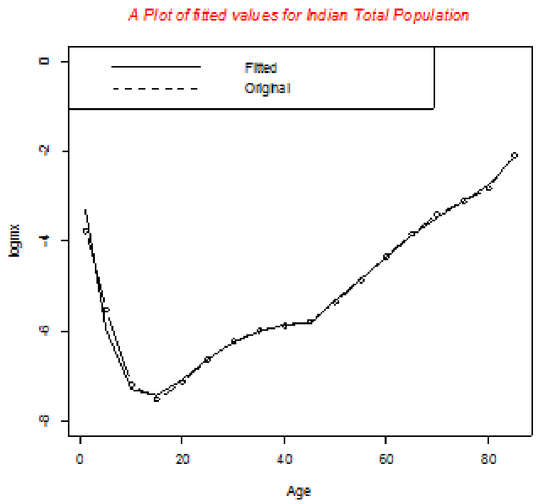
(o) (p)

Figures (o-p): Comparison of Observed and Fitted values of Age-Specific Mortality (in log scales) for (o) Urban Female (Adjusted R^2=0.969) and (p) Male (Adjusted R^2=0.9739) Population during the Year 2012-16.



(q) **(r)**

Figures (q-r): Comparison of Observed and Fitted Values of Age-Specific Mortality (in log scales) for (q) Urban Total (Adjusted R^2=0.9869) and (r) Indian Total (Adjusted R^2=0.9904) Population during the Year 2012-16.



(s)

Figure (s): Comparison of Observed and Fitted Values of Age-Specific Mortality (in log scales) for Total Indian (Adjusted R^2=0.9897) Population during the Year 2012-16.

CONSTRUCTION OF LIFE TABLES

We first obtained the age-specific death rates (${}_nm_x$) for each age-group for India from SRS data of India 2012-16. We use the values of ${}_na_x$, the average number of years lived in the x to $x+n$ age interval by those dying in the interval (see Chiang, 1984 and Schoen, 1978). Using the set of values of ${}_na_x$ for different age groups of India (2012-16), we constructed the life tables for total, males, females including rural and urban populations. For comparisons, the values of ${}_nq_x$ and e_x^0 of subpopulations are given in the Tables 1-3.Chiang noted that the value of the fraction ${}_na_x$ depends on the mortality pattern over an entire age interval but not on the mortality rate for any single year, we take ${}_na_x = \frac{n}{2}$. When the mortality rate decreases with age in an interval, the fraction ${}_na_x > \frac{n}{2}$ is considered, when reverse prevails ${}_na_x < \frac{n}{2}$ is considered. Therefore we considered ${}_n a_x$ =0.365i for age group 1-5 and ${}_n a_x$ =0.641i for the age group 15-20, where i is the age interval. This is due to the difference in mortality pattern in the above mentioned groups.

Different columns of the life tables are calculated by using their interrelationships.

Age group $(x, x+n)$

Age interval or period of life between two exact ages stated in years

$${}_n q_x = \frac{n * {}_n m_x}{(1 + (n - {}_n a_x) * {}_n m_x)}$$

Proportion of persons alive at the beginning of the age interval who die during the age interval

$l_x = l_x - \left(l_x * {}_n q_x\right)$ Of the starting number of newborns in the life table (called the radix of the life table, usually set at 100,000) the number living at the beginning of the age interval (or the number surviving to the beginning of the age interval)

$${}_n d_x = l_x * {}_n q_x$$

The number of persons in the cohort who die in the age interval (x, x+ n)

$${}_n L_x = \frac{{}_n d_x}{{}_n m_x}$$

Number of years of life lived by the cohort within the indicated age interval (x, x+ n) (or person-years of life in the age interval)

$$.T_x = L_x + L_{x+1} + \cdots + L_n$$

Total person-years of life contributed by the cohort after attaining age x

$$.e_x^0 = \frac{T_x}{l_x}$$

Average number of years of life remaining for a person alive at the beginning of age interval x

Table 1: Estimated Mortality Rate (${}_n q_x$) and Life Expectancy (e_x^0) for Total, Female and Male Indian Population

Age Group	Total Population		Female Population		Male Population	
	${}_n q_x$	e_x^0	${}_n q_x$	e_x^0	${}_n q_x$	e_x^0
0-1	0.04014	67.80899	0.04137	69.06721	0.03908	66.63974
1-5	0.00770	69.62376	0.00922	71.02625	0.00635	68.3296
5-10	0.00379	66.13884	0.00394	67.65687	0.00364	64.74578
10-15	0.00315	61.38095	0.00295	62.91461	0.00334	59.97318
15-20	0.00484	56.56701	0.00489	58.09336	0.00479	55.16578
20-25	0.00698	51.83312	0.00638	53.36973	0.00757	50.4224
25-30	0.00777	47.17988	0.00668	48.69636	0.00886	45.78794
30-35	0.00975	42.52976	0.00742	44.00702	0.01193	41.1749
35-40	0.01341	37.9239	0.0098	39.31731	0.01691	36.64186
40-45	0.01834	33.40539	0.01376	34.68169	0.02256	32.22913
45-50	0.02623	28.98278	0.01879	30.13069	0.03322	27.9153
50-55	0.04254	24.69614	0.03609	25.65981	0.04819	23.7886

Table 1 Contd.,

55-60	0.06241	20.68231	0.04957	21.52694	0.07601	19.86644
60-65	0.09228	16.89261	0.08078	17.5193	0.10323	16.29505
65-70	0.13633	13.35578	0.12197	13.83918	0.15047	12.88305
70-75	0.20462	10.06936	0.18581	10.41434	0.22384	9.72211
75-80	0.29153	7.01666	0.26625	7.22051	0.31827	6.80492
80+	0.44991	3.87523	0.42664	3.9334	0.47412	3.8147

Table 2: Estimated Mortality Rate ($_nq_x$) and Life Expectancy (e_x^0) for Indian Total, Female and Male Rural Population

Age Group	Total Population		Female Population		Male Population	
	$_nq_x$	e_x^0	$_nq_x$	e_x^0	$_nq_x$	e_x^0
0-1	0.0447	66.61674	0.04575	67.90930	0.04379	65.40127
1-5	0.00926	68.71044	0.01132	70.14113	0.00747	67.37345
5-10	0.00419	65.32221	0.00439	66.90719	0.00404	63.85602
10-15	0.00339	60.58654	0.00315	62.19118	0.00359	59.1049
15-20	0.00509	55.78413	0.00514	57.37981	0.00504	54.30884
20-25	0.00752	51.06004	0.00683	52.66668	0.00817	49.57457
25-30	0.00876	46.42798	0.00777	48.01168	0.0097	44.96234
30-35	0.01084	41.81619	0.00856	43.36807	0.01297	40.37825
35-40	0.01475	37.24705	0.01109	38.72092	0.01829	35.87599
40-45	0.0205	32.76724	0.01593	34.12712	0.02476	31.49781
45-50	0.02881	28.4007	0.02104	29.63909	0.03617	27.23403
50-55	0.04762	24.16904	0.0411	25.22237	0.05329	23.16223
55-60	0.06804	20.25251	0.05299	21.19629	0.08484	19.3253
60-65	0.10096	16.54858	0.08865	17.24244	0.11282	15.88509
65-70	0.14446	13.1262	0.12781	13.67649	0.16108	12.58723
70-75	0.2144	9.92046	0.19275	10.31428	0.23678	9.52407
75-80	0.29839	6.94559	0.26953	7.18012	0.32898	6.70320
80+	0.46549	3.83628	0.4372	3.907	0.49444	3.7639

Table 3: Estimated Mortality Rate ($_nq_x$) and Life Expectancy (e_x^0) for Indian Total, Female and Male Urban Population

Age Group	Total Population		Female Population		Male Population	
	$_nq_x$	e_x^0	$_nq_x$	e_x^0	$_nq_x$	e_x^0
0-1	0.0263	70.91088	0.02797	72.11978	0.02483	69.8101
1-5	0.00303	71.81270	0.00295	73.18063	0.00315	70.57489
5-10	0.00265	68.02135	0.00275	69.38778	0.0026	66.78791
10-15	0.0025	63.19543	0.00235	64.57223	0.0026	61.95549
15-20	0.00409	58.34755	0.00414	59.71845	0.00404	57.11048
20-25	0.00568	53.57959	0.00529	54.95903	0.00608	52.33465
25-30	0.00573	48.87137	0.00439	50.23801	0.00713	47.6395
30-35	0.00757	44.13861	0.00504	45.4485	0.00995	42.96366
35-40	0.01074	39.45622	0.00713	40.66606	0.0141	38.37032
40-45	0.01391	34.85744	0.00931	35.94014	0.01809	33.88332
45-50	0.02099	30.31388	0.01421	31.25439	0.02729	29.46151
50-55	0.03294	25.91021	0.02687	26.66888	0.03844	25.21793
55-60	0.05085	21.70761	0.04186	22.33623	0.05929	21.12612
60-65	0.07291	17.73664	0.06267	18.20285	0.08226	17.30006
65-70	0.11672	13.93491	0.10739	14.25275	0.12557	13.62664
70-75	0.1804	10.44597	0.16835	10.66672	0.19238	10.22445
75-80	0.27416	7.19493	0.2579	7.31990	0.29117	7.06446
80+	0.41269	3.96828	0.40202	3.99495	0.42423	3.93943

CONCLUSIONS

We use the polynomial regression model to fit mortality data where mortalities are available in age interval. In case of the higher order polynomials, the problem of multicollinearity is resolved by centering explanatory variable. We observe from the figures that fitting is very good for all the heterogeneous subpopulations except for urban population. Thevalidity of the model is done by the adjusted - R^2. For all the subpopulations polynomial approximation is the suitable choice of fitting model to mortality data. Once the mortality data by age interval is obtained, rest of the columns of life tables are constructed using their interrelationships. The fitting for both the cases (lower limit and upper limit) and for all subpopulations is good. We consider a unique polynomial approximation using these two best fit polynomials. This polynomial approximation is good for all subpopulations except rural male, urban male and female subpopulations.

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